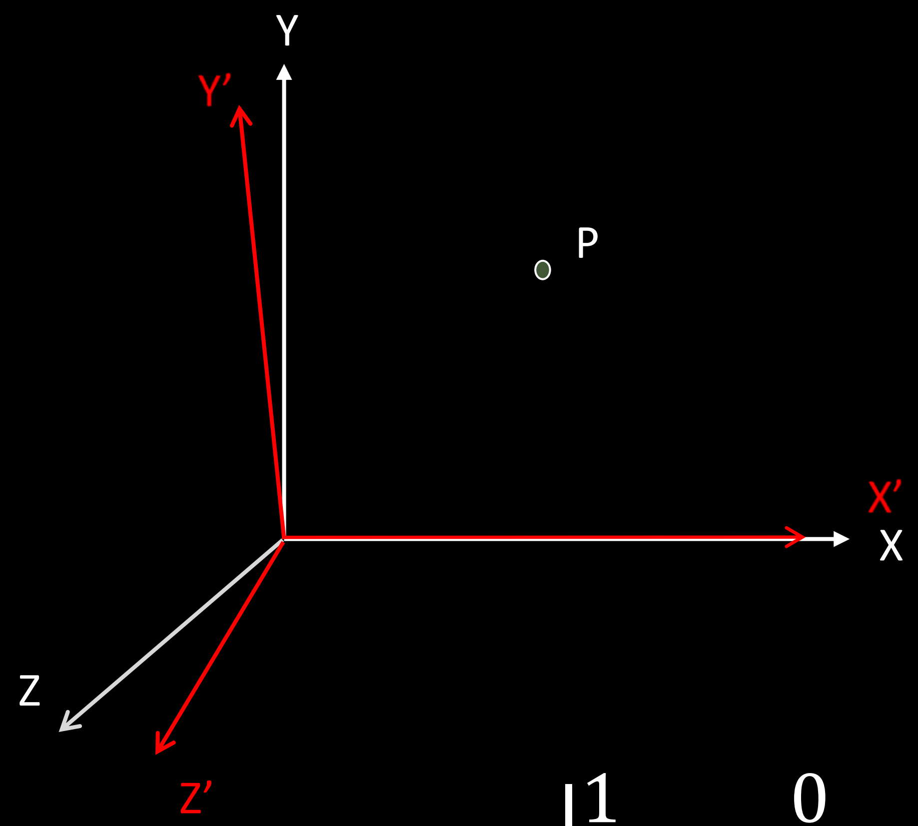
Tonight's Lecture

- Segmental Angles
- Joint Angles: Euler
- Joint Angles: Helical
- Exporting Data from GaitProject to Visual3d

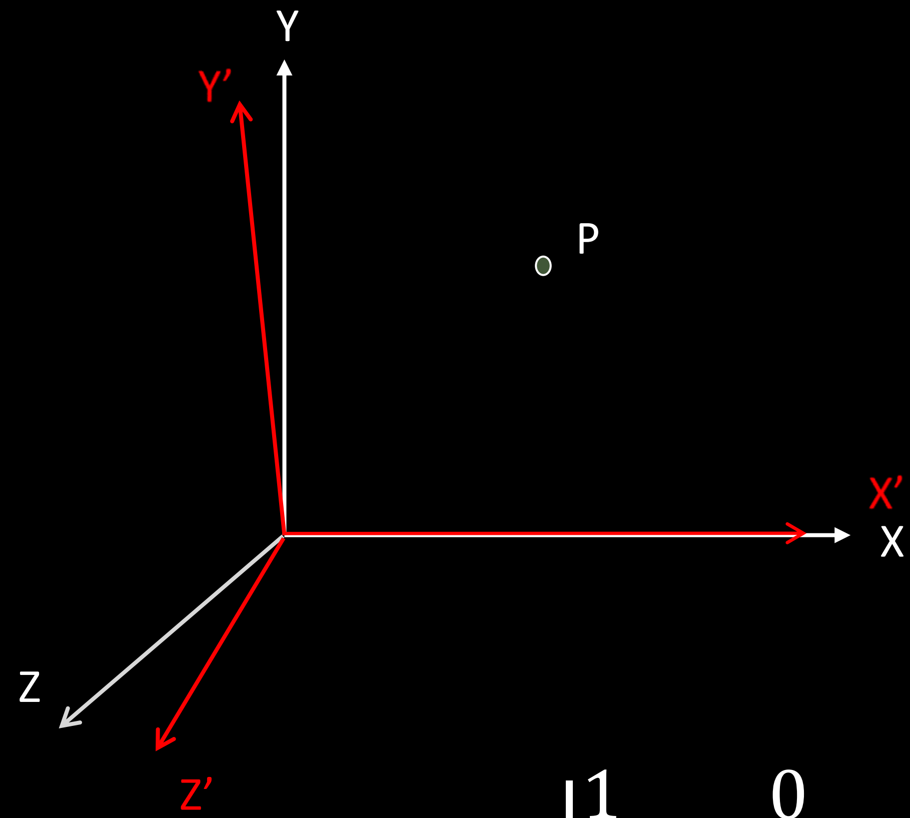
Segmental Angles

X Rotation from Global to Local



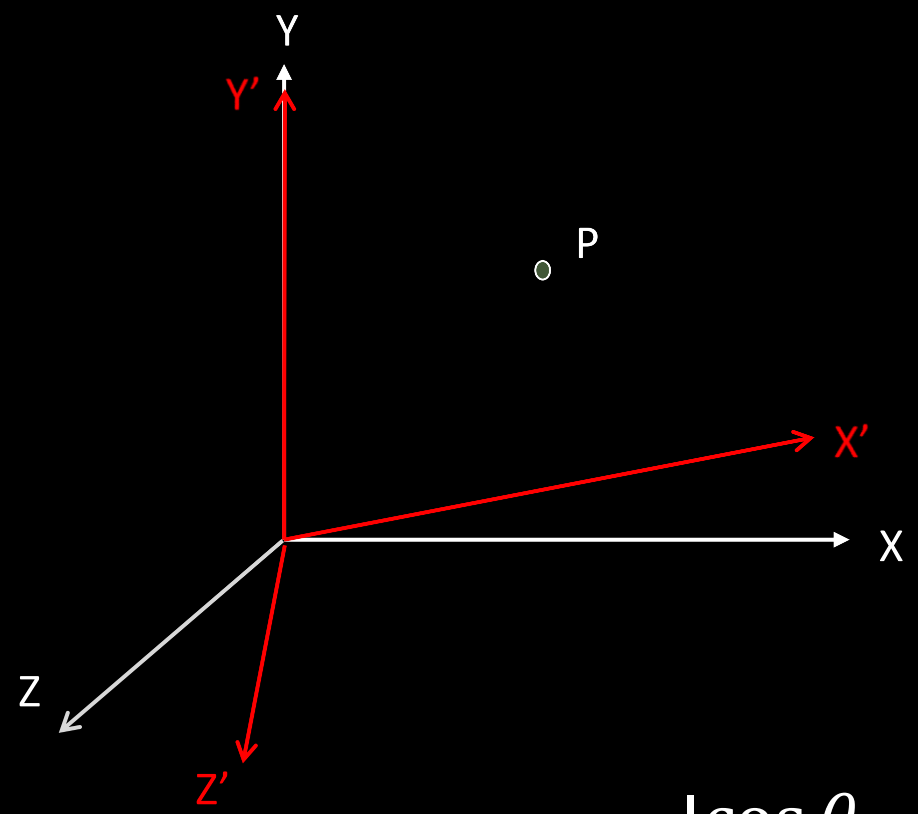
$$R_x = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & \sin\theta_x \\ 0 & -\sin\theta_x & \cos\theta_x \end{vmatrix}$$

X Rotation Local to Global



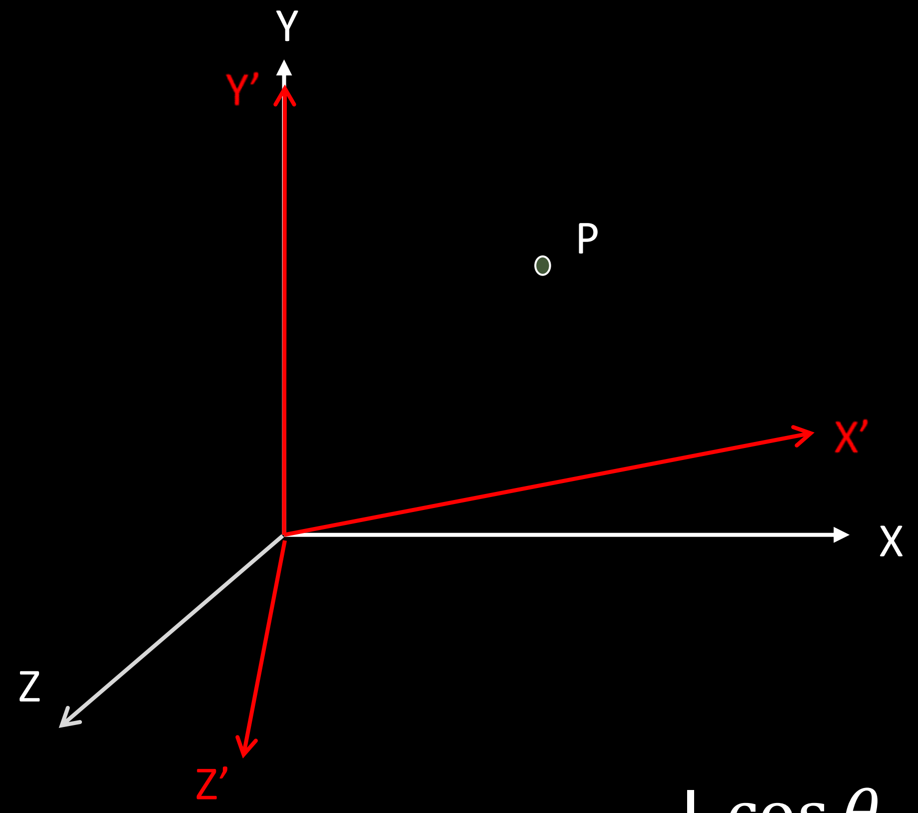
$$R_x^t = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{vmatrix}$$

Y Rotation from Global to Local



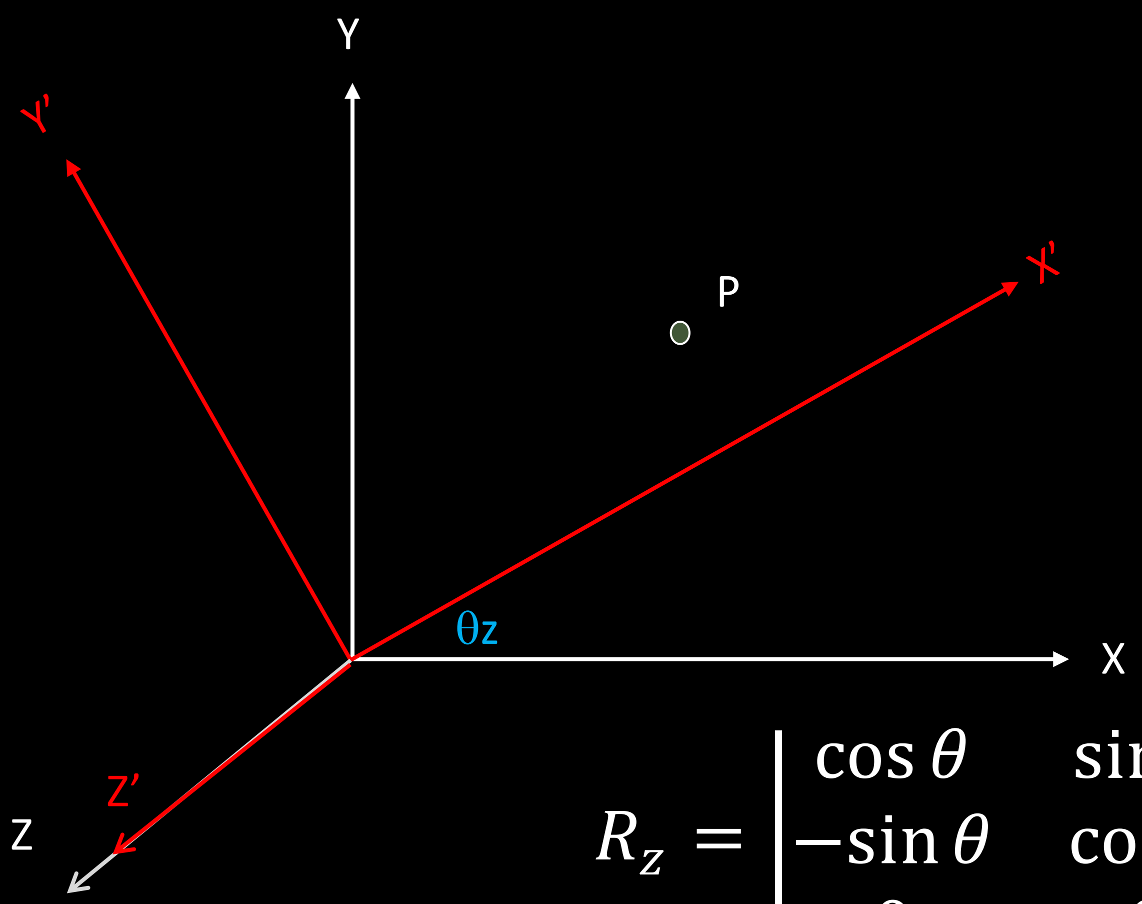
$$R_y = \begin{vmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{vmatrix}$$

Y Rotation from Local to Global



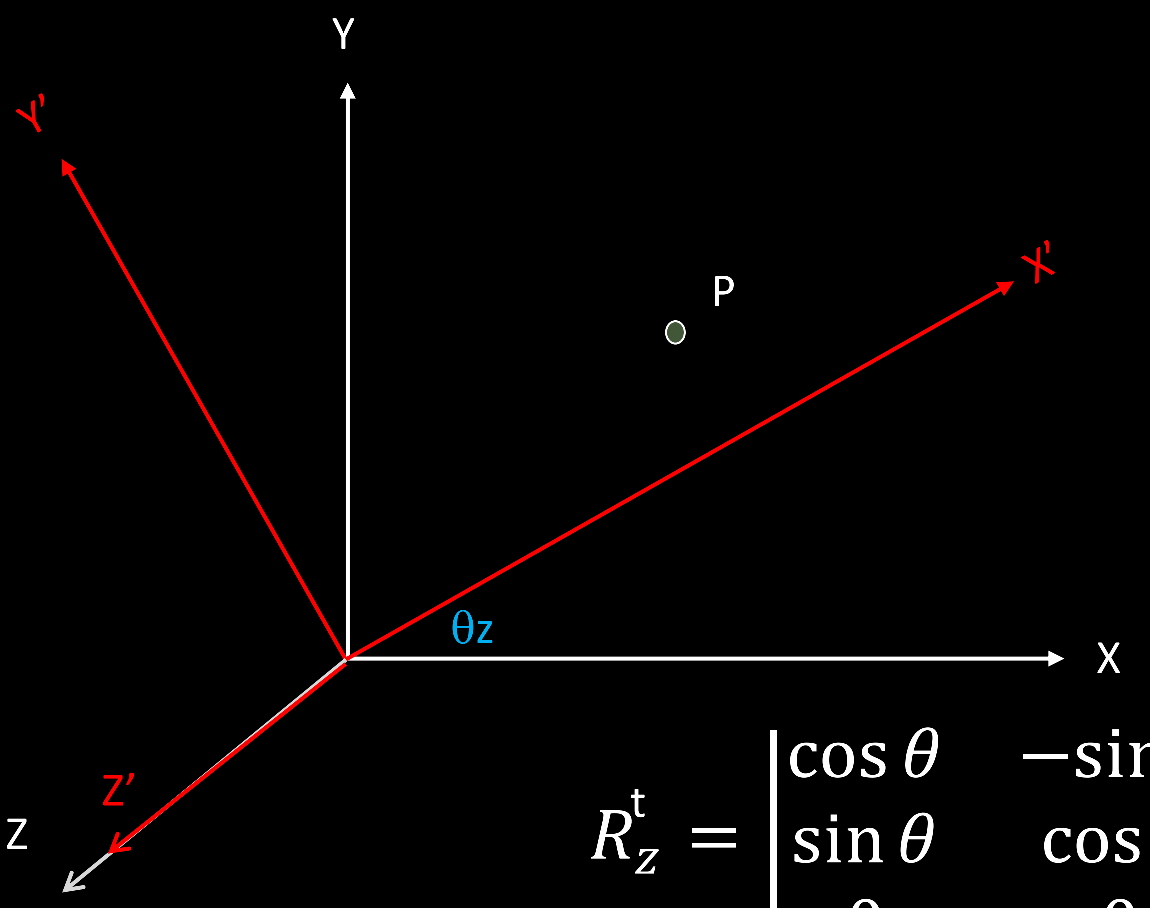
$$R_y^t = \begin{vmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{vmatrix}$$

Z Rotation from Global to Local



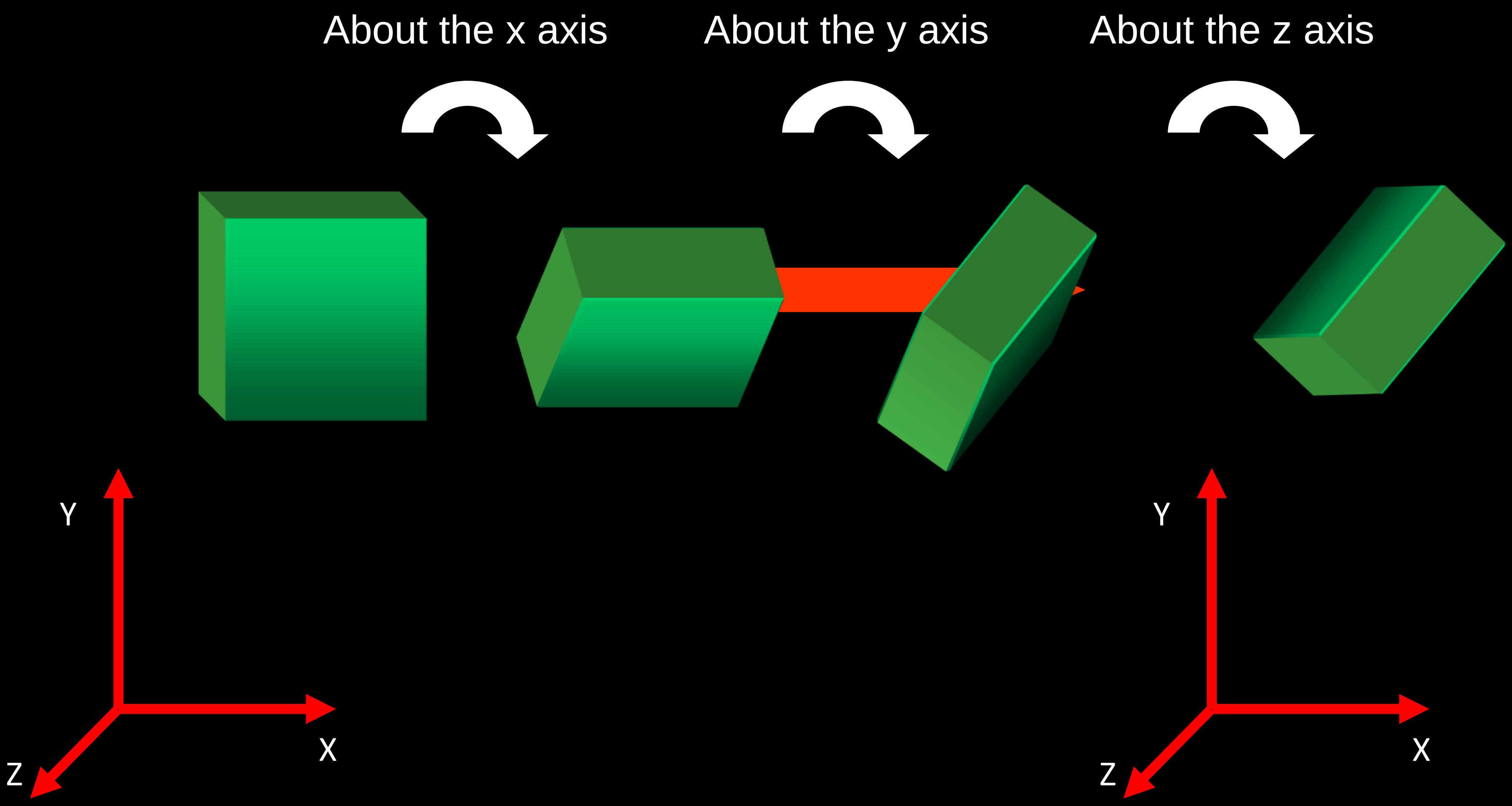
$$R_z = \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Z Rotation from Local to Global

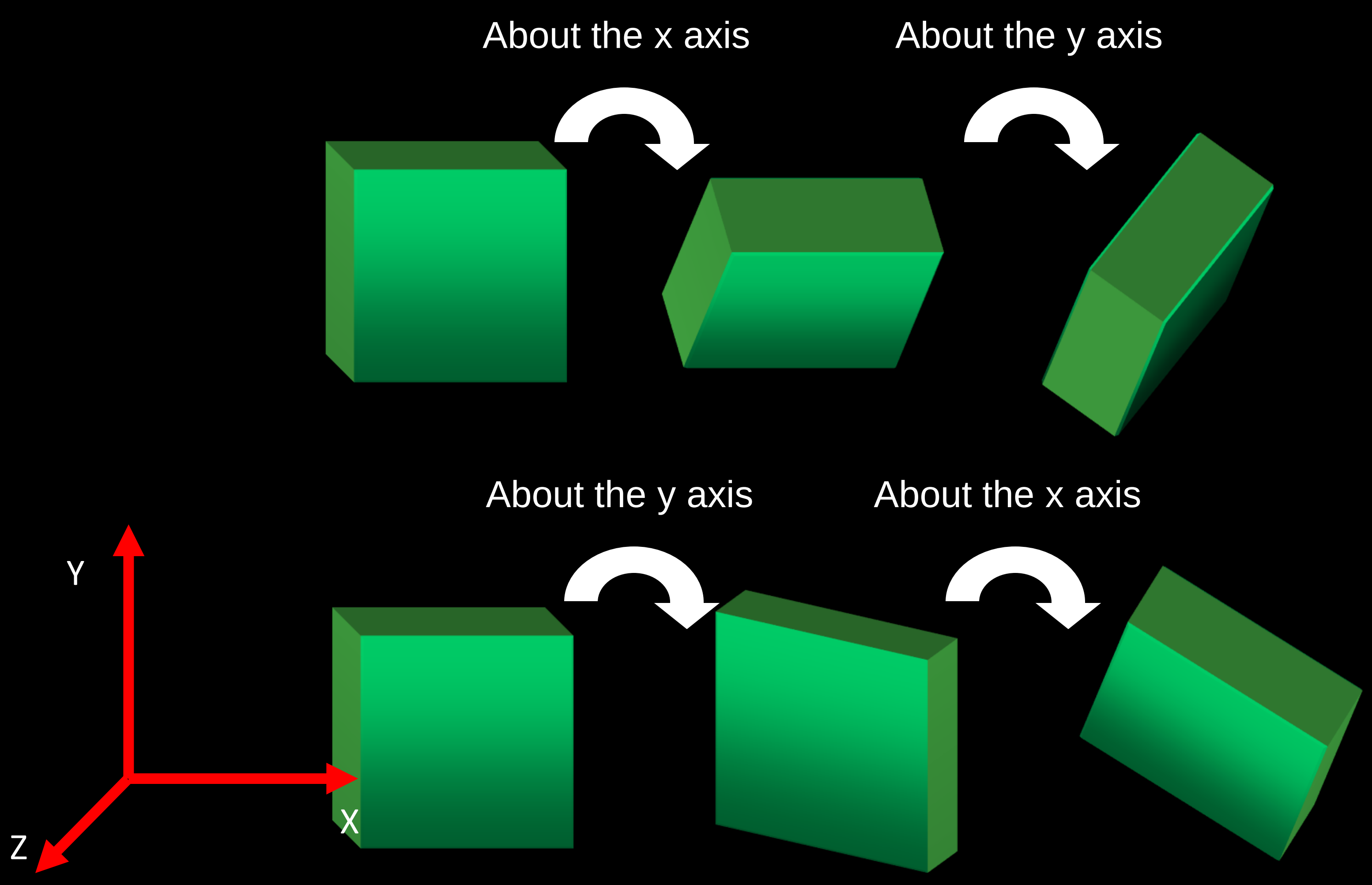


$$R_z^t = \begin{vmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Orientation of Bodies in 3D



The final orientation depends on the rotation sequence



Rotation Matrices 3D

$$(\theta_x , \theta_y , \theta_z)$$

$$R = R_z R_y R_x$$

- Rotation's depend on the order they are applied

Rotation Matrices 3D

$$R = R_z R_y R_x$$

$$R_y R_x = \begin{vmatrix} \cos\theta_y & 0 & -\sin\theta_y \\ 0 & 1 & 0 \\ \sin\theta_y & 0 & \cos\theta_y \end{vmatrix} \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & \sin\theta_x \\ 0 & -\sin\theta_x & \cos\theta_x \end{vmatrix}$$

$$R_y R_x = \begin{vmatrix} \cos\theta_y & \sin\theta_y \sin\theta_x & -\sin\theta_y \cos\theta_x \\ 0 & \cos\theta_x & \sin\theta_x \\ \sin\theta_y & -\cos\theta_y \sin\theta_x & \cos\theta_y \cos\theta_x \end{vmatrix}$$

Rotation Matrices 3D

$$R = R_z R_y R_x$$

$$R_z R_y R_x = \begin{vmatrix} \cos\theta_z & \sin\theta_z & 0 \\ -\sin\theta_z & \cos\theta_z & 0 \\ 0 & 0 & 1 \end{vmatrix} \begin{vmatrix} \cos\theta_y & \sin\theta_y \sin\theta_x & -\sin\theta_y \cos\theta_x \\ 0 & \cos\theta_x & \sin\theta_x \\ \sin\theta_y & -\cos\theta_y \sin\theta_x & \cos\theta_y \cos\theta_x \end{vmatrix}$$

$$R_z R_y R_x = \begin{vmatrix} \cos\theta_z \cos\theta_y & \cos\theta_z \sin\theta_y \sin\theta_x + \sin\theta_z \cos\theta_x & -\cos\theta_z \sin\theta_y \cos\theta_x + \sin\theta_z \sin\theta_x \\ -\sin\theta_z \cos\theta_y & -\sin\theta_z \sin\theta_y \sin\theta_x + \cos\theta_z \cos\theta_x & \sin\theta_z \sin\theta_y \cos\theta_x + \cos\theta_z \sin\theta_x \\ \sin\theta_y & -\cos\theta_y \sin\theta_x & \cos\theta_y \cos\theta_x \end{vmatrix}$$

Euler Angles: Simple Method

$$R_z R_y R_x = \begin{vmatrix} \cos\theta_z \cos\theta_y & \cos\theta_z \sin\theta_y \sin\theta_x + \sin\theta_z \cos\theta_x & -\cos\theta_z \sin\theta_y \cos\theta_x + \sin\theta_z \sin\theta_x \\ -\sin\theta_z \cos\theta_y & -\sin\theta_z \sin\theta_y \sin\theta_x + \cos\theta_z \cos\theta_x & \sin\theta_z \sin\theta_y \cos\theta_x + \cos\theta_z \sin\theta_x \\ \sin\theta_y & -\cos\theta_y \sin\theta_x & \cos\theta_y \cos\theta_x \end{vmatrix}$$

$$R = \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix}$$

$$\theta_y = asin(R_{31})$$

$$\theta_x = asin(-\frac{R_{32}}{cos\theta_y})$$

$$\theta_z = asin(-\frac{R_{21}}{cos\theta_y})$$

Euler Angles: Robust Method
(handles angles from -180 to +180)

$$R_z R_y R_x = \begin{vmatrix} \cos\theta_z \cos\theta_y & \cos\theta_z \sin\theta_y \sin\theta_x + \sin\theta_z \cos\theta_x & -\cos\theta_z \sin\theta_y \cos\theta_x + \sin\theta_z \sin\theta_x \\ -\sin\theta_z \cos\theta_y & -\sin\theta_z \sin\theta_y \sin\theta_x + \cos\theta_z \cos\theta_x & \sin\theta_z \sin\theta_y \cos\theta_x + \cos\theta_z \sin\theta_x \\ \sin\theta_y & -\cos\theta_y \sin\theta_x & \cos\theta_y \cos\theta_x \end{vmatrix}$$

$$R = \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix}$$

$$\theta_z = atan2(-R_{21}, R_{11})$$

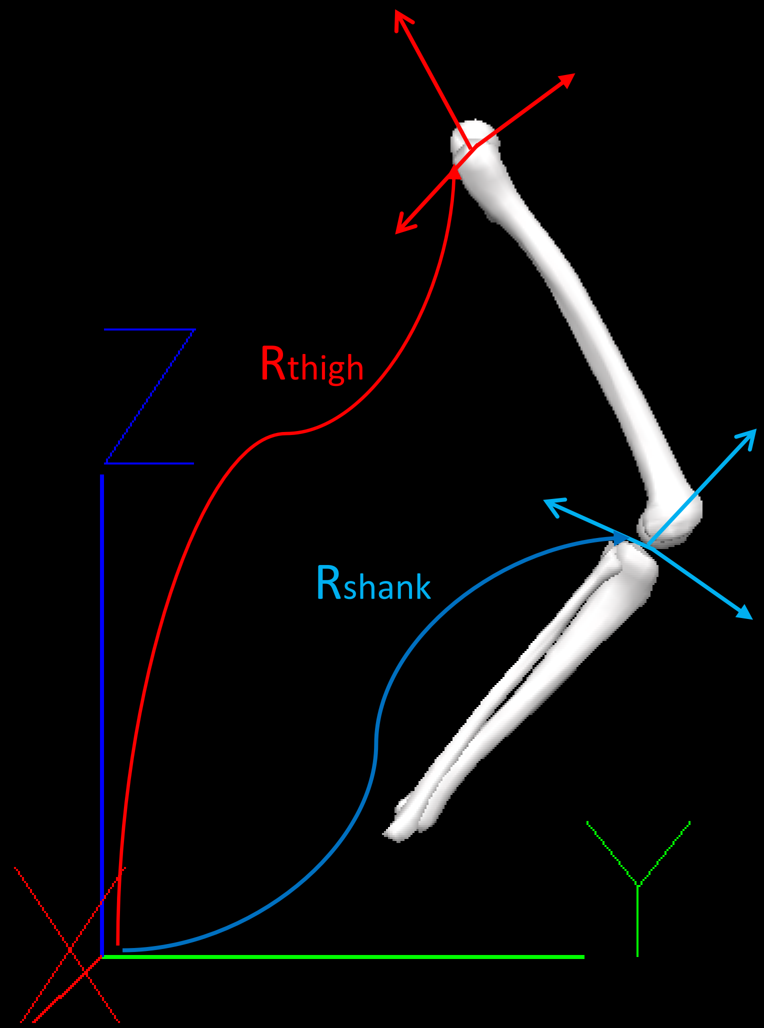
$$\theta_x = atan2(-R_{32}, R_{33})$$

$$\theta_y = atan2(R_{31}, \sqrt{{R_{11}}^2 + {R_{12}}^2})$$

Atan2(x,y) is the arc tangent of x/y in the interval [-pi,+pi] radians

Joint Angles

Relative Motion: Joint Angles



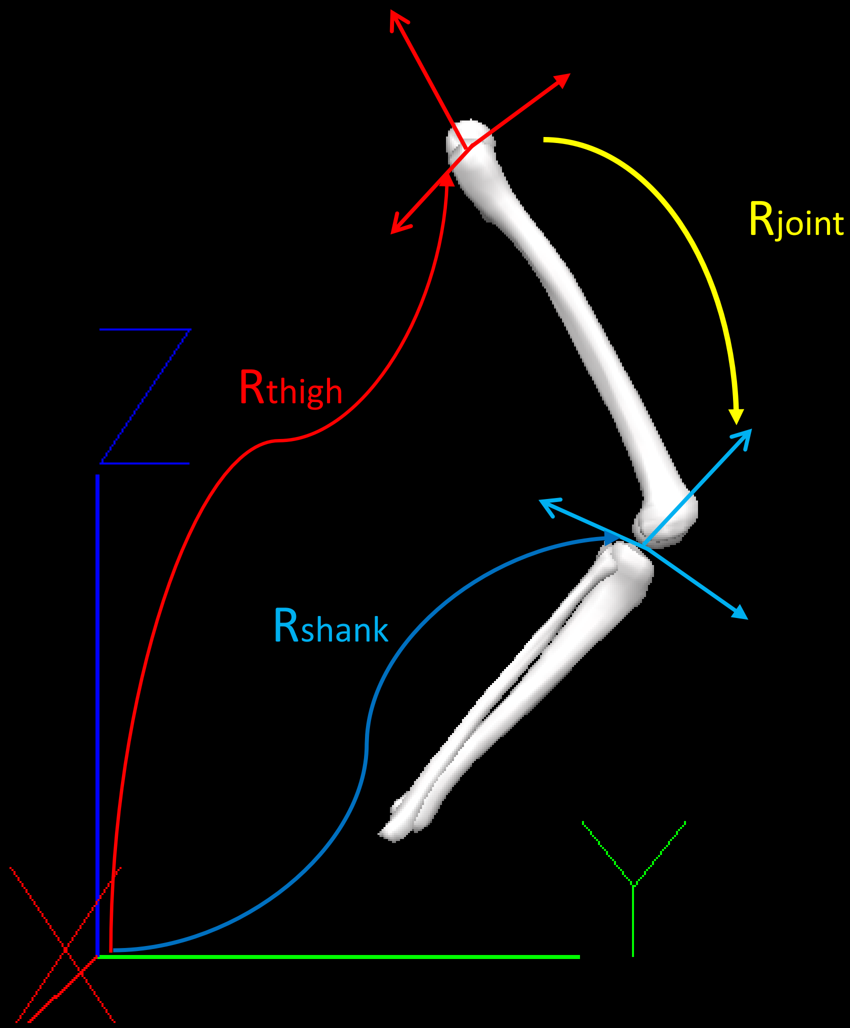
Joint Angles

$R_{joint} * R_{thigh} = R_{shank}$

$R_{joint} = R_{shank} * R_{thigh}'$

General form:

$R_{joint} = R_{distal} * R_{proximal}'$



Joint Angles

$R_{joint} = R_{distal} * R_{proximal}'$

$$R_{joint} = \begin{vmatrix} \cos\theta_z \cos\theta_y & \cos\theta_z \sin\theta_y \sin\theta_x + \sin\theta_z \cos\theta_x & -\cos\theta_z \sin\theta_y \cos\theta_x + \sin\theta_z \sin\theta_x \\ -\sin\theta_z \cos\theta_y & -\sin\theta_z \sin\theta_y \sin\theta_x + \cos\theta_z \cos\theta_x & \sin\theta_z \sin\theta_y \cos\theta_x + \cos\theta_z \sin\theta_x \\ \sin\theta_y & -\cos\theta_y \sin\theta_x & \cos\theta_y \cos\theta_x \end{vmatrix}$$

$$R = \begin{vmatrix} R_{00} & R_{01} & R_{02} \\ R_{10} & R_{11} & R_{12} \\ R_{20} & R_{21} & R_{22} \end{vmatrix}$$

abduction

$$\theta_y = asin(R_{20})$$

flexion

$$\theta_x = asin(-\frac{R_{21}}{cos\theta_y})$$

Internal/external

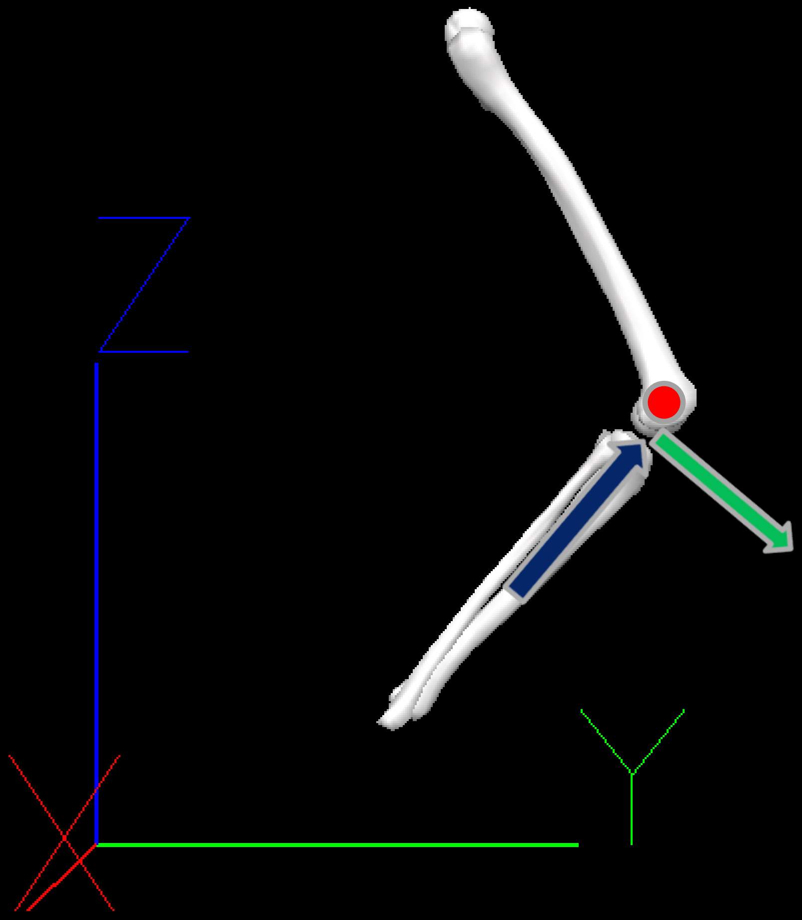
$$\theta_z = asin(-\frac{R_{10}}{cos\theta_y})$$

Lower Extremity: Joint Coordinate System

Red – flexion extension (thigh)

Green – ab/adduction (float)

Blue – internal/external (shank)



Joint Angles: Gimbal Lock

$R_{joint} = R_{distal} * R_{proximal}'$

$$R = \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix}$$

ab/adduction

$$\theta_y = asin(-R_{31})$$

flexion

$$\theta_x = asin(-\frac{R_{32}}{cos\theta_y})$$

Internal/external

$$\theta_z = asin(-\frac{R_{21}}{cos\theta_y})$$

What happens if the ab/aduction angle approaches 90 degrees?

Joint Angles: Gimbal Lock

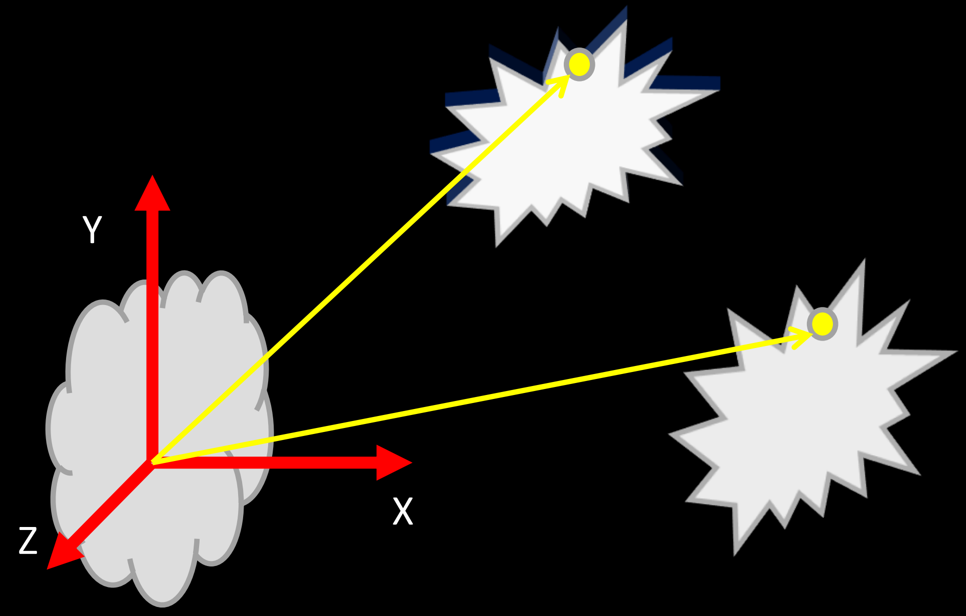
Solutions:

- Use alternative Euler Sequence (for example ZYZ is the ISB recommended sequence for the shoulder)

- Helical Angles

Helical Angles*

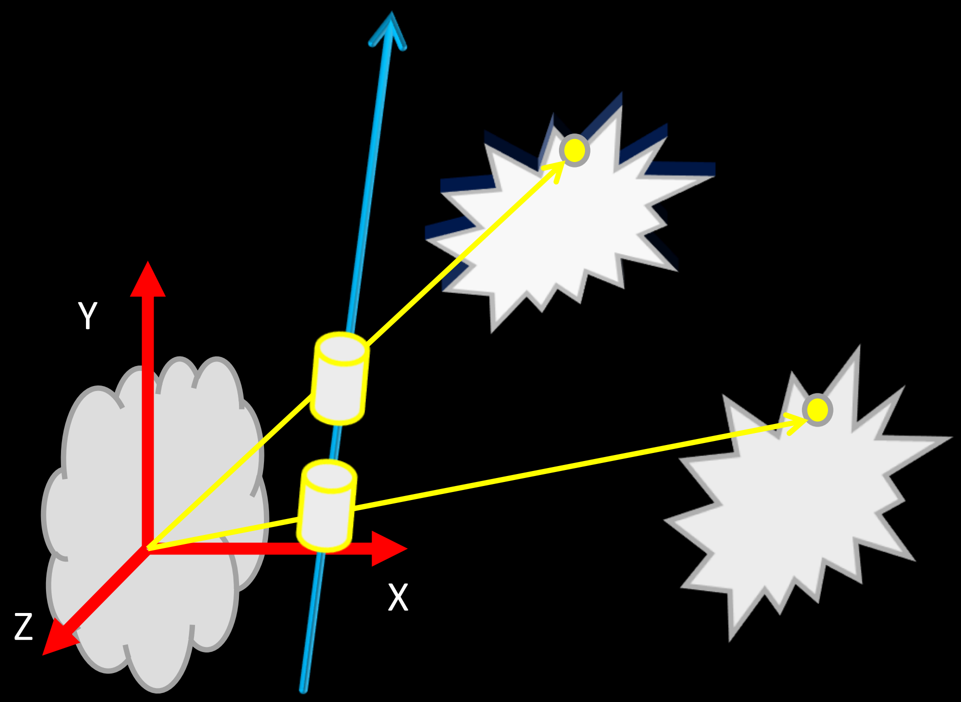
"The displacement of a moving body from position 1 to position 2 can always be represented as a rotation about, and a translation along, a unique axis located Somewhere in the fixed body".



*From: Kinzel, Hall and Hillberry, J Biomech, 5, 1972

Helical Angles*

“The displacement of the moving body is completely defined if the location and inclination of the screw axis, the rotation angle ϕ, and the translation magnitude K are know ”.



*From: Kinzel, Hall and Hillberry, J Biomech, 5, 1972

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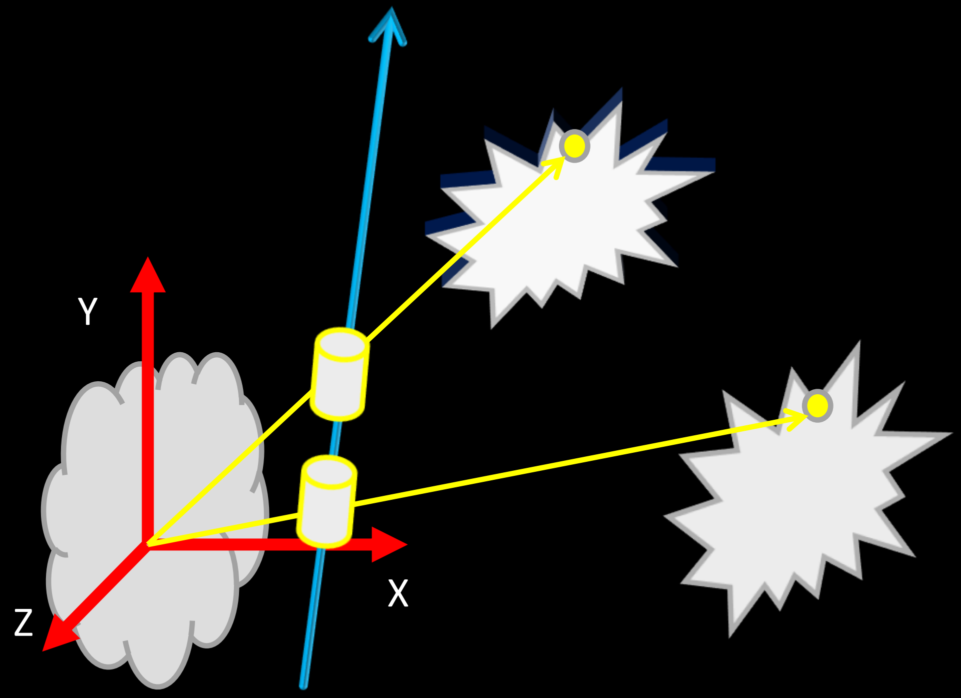


Rotation:

$$R_{motion} = \begin{vmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{vmatrix}$$

*From: Kinzel, Hall and Hillberry, J Biomech, 5, 1972

Helical Angles

Superfluous Math Warning!!

Helical Angles: Finding the magnitude of the rotation

$$R = \begin{vmatrix} {u_x}^2(1-cos\emptyset)+\ cos\emptyset & u_xu_y(1-\cos\emptyset)+u_zsin\emptyset & u_zu_x(1-\cos\emptyset)-u_ysin\emptyset \\ u_xu_y(1-\cos\emptyset)-u_zsin\emptyset & {u_y}^2(1-cos\emptyset)+\ cos\emptyset & u_yu_z(1-\cos\emptyset)-u_xsin\emptyset \\ u_zu_x(1-\cos\emptyset)-u_ysin\emptyset & u_yu_z(1-\cos\emptyset)+u_xsin\emptyset & {u_z}^2(1-cos\emptyset)+\ cos\emptyset \end{vmatrix} \ ^*$$

$$Trace(R) = \ {u_x}^2(1-cos\emptyset)+\ cos\emptyset+\ {u_y}^2(1-cos\emptyset)+\ cos\emptyset+\ {u_z}^2(1-cos\emptyset)+\ cos\emptyset$$

$$Trace(R) = (1-cos\emptyset)\left({u_x}^2+{u_y}^2+{u_z}^2\right)+\ 3cos\emptyset$$

$$Trace(R) = 2cos\emptyset+1$$

$$cos\emptyset = \frac{Trace(R)-1}{2}$$

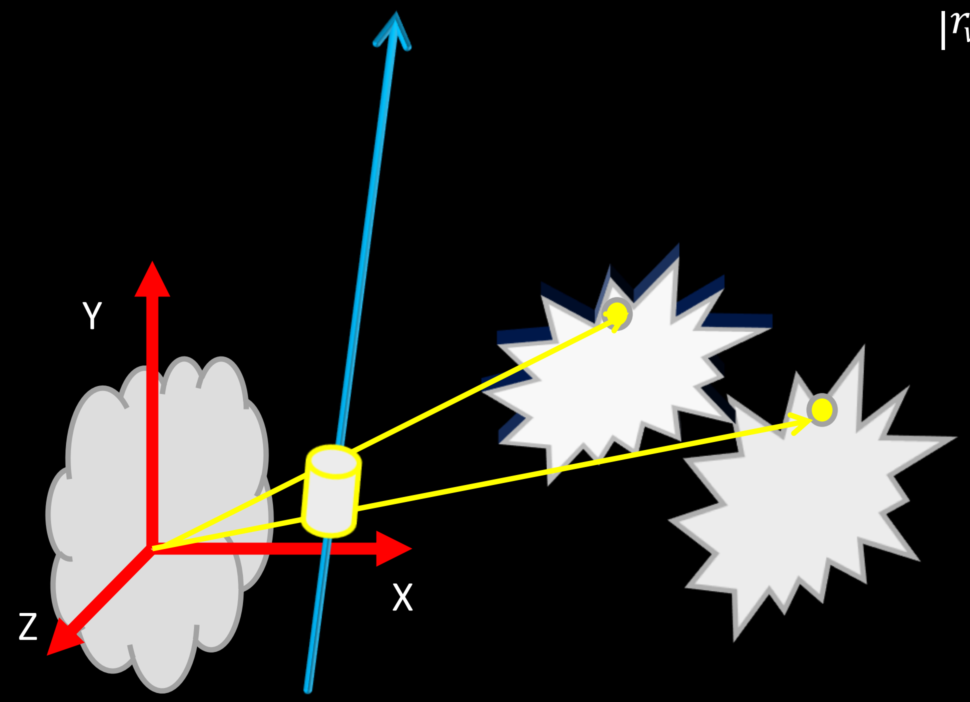
$$\boldsymbol{\emptyset = acos\left[\frac{Trace(R)-\ 1}{2}\right]}$$

*From: Hall, 1961

Helical Angles: Finding the direction of the axis

A pure rotation of a point W about the helical axis is given by:

$$|r_{w_2}| = R|r_{w_1}|$$



Helical Angles: Finding the direction of the axis

From the previous slide: a pure rotation of a point W about the helical axis is given by:

$$|r_{w_2}| = R|r_{w_1}|$$

But if W is on the axis of rotation W_1 and W_2 are the same point then the vector r_w lies on the axis of rotation and:

$$|r_w| = R|r_w|$$

$$[R - I]|r_w| = |0|$$

Let u be r_w with a magnitude of unity the u become the components of the helical axis and….

Helical Angles: Finding the direction of the axis*

$$[R - I]|u| = |0|$$

or:

$$\begin{vmatrix} r_{11} - 1 & r_{12} & r_{13} \\ r_{21} & r_{22} - 1 & r_{23} \\ r_{31} & r_{32} & r_{33} - 1 \end{vmatrix} \begin{vmatrix} u_x \\ u_y \\ u_z \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix}$$

and from algebra:

$$u_y = \frac{u_x[r_{23}r_{31} - r_{21}(r_{33} - 1)]}{(r_{22} - 1)(r_{33} - 1) - r_{23}r_{32}}$$

$$u_z = \frac{u_x[r_{21}r_{32} - r_{31}(r_{22} - 1)]}{(r_{22} - 1)(r_{33} - 1) - r_{23}r_{32}}$$

$${u_x}^2 = 1 - {u_y}^2 - {u_z}^2$$

*From: Kinzel, Hall and Hillberry, J Biomech, 5, 1972

End Superfluous Math Warning!!

Helical Angles

Advantages:

- No problems with gimbal lock
- Provides a vector quantity
- Provide a general solution usable at all joints

Disadvantages:

- Can be difficult for anatomical interpretation