Introduction to Force Platforms – Part2

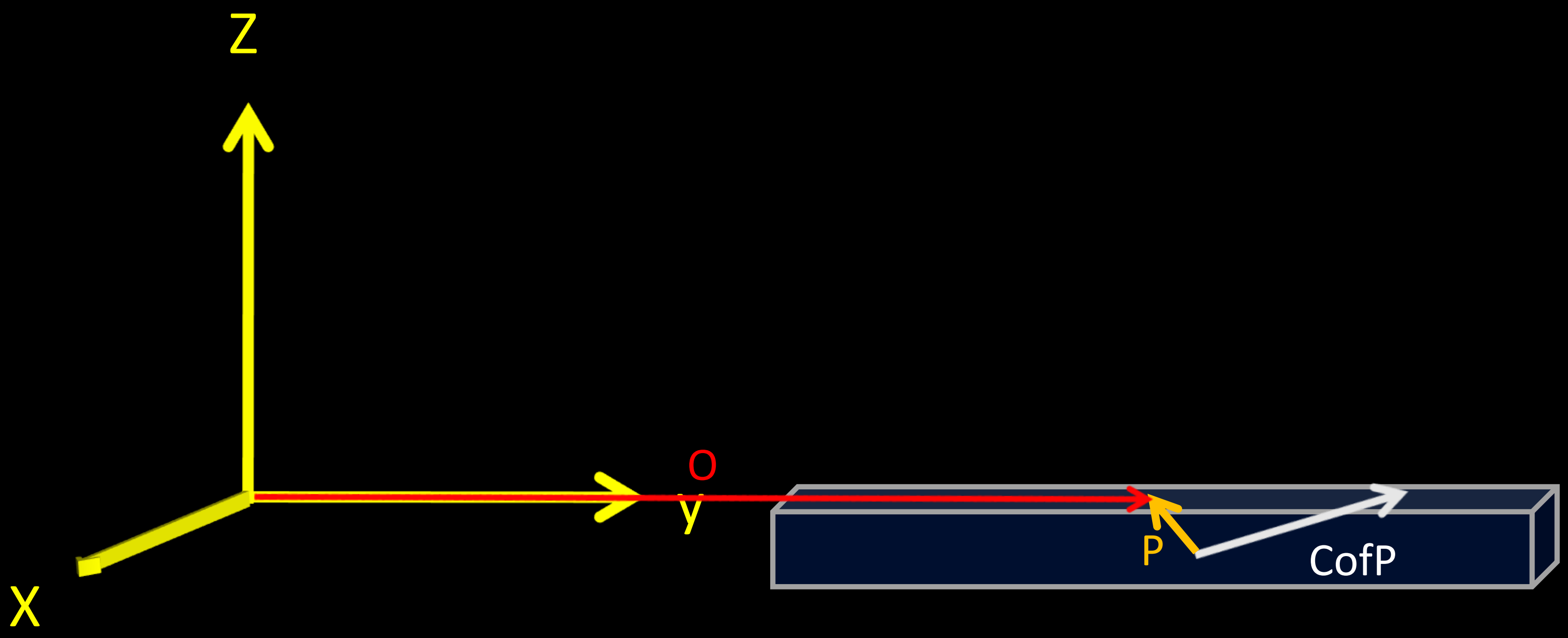
Review: For Force and Free Moment only the Rotational Transformation is required

$$\overline{F} = R^t \overline{F}'$$

$$\overline{\tau} = R^t \overline{\tau}'$$

Review: Center of Pressure from Local to Global

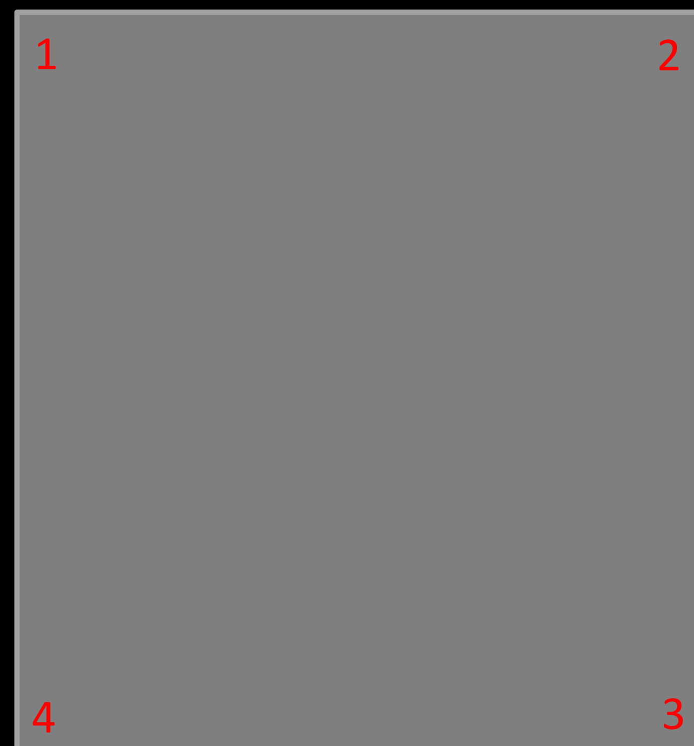
$$CofP_{global} = \bar{O} - R^t \bar{P}_{local} + R^t \overline{CofP}_{local}$$



Assignment: 5

C3d File format specifies that R and **O** are supplied indirectly via the location of the four corners

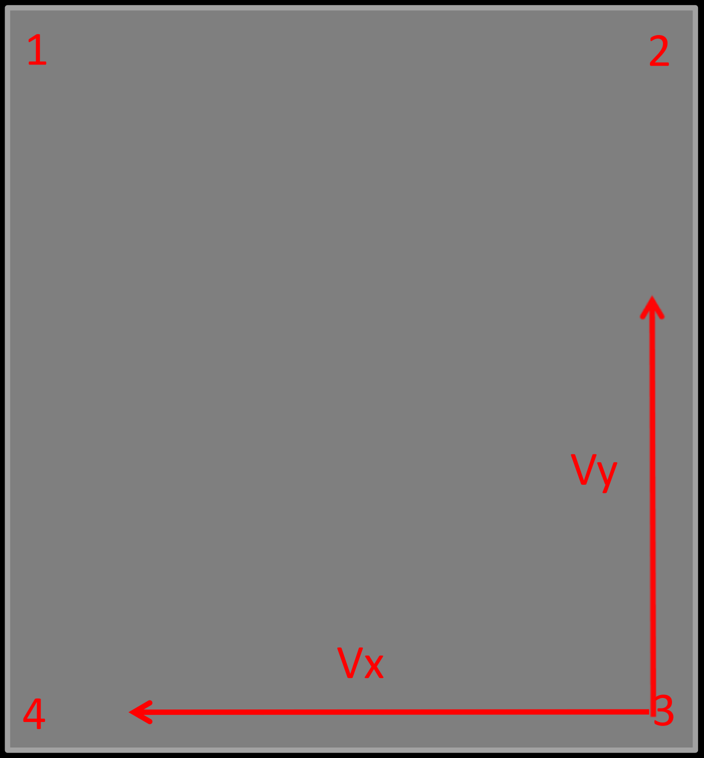
$$\bar{O} = \frac{\bar{P}_1 + \bar{P}_2 + \bar{P}_3 + \bar{P}_4}{4}$$



C3d File format specifies that **R** and O are supplied indirectly via the location of the four corners

$$\bar{V}_x = \bar{P}_4 - \bar{P}_3$$

$$\bar{V}_y = \bar{P}_2 - \bar{P}_3$$

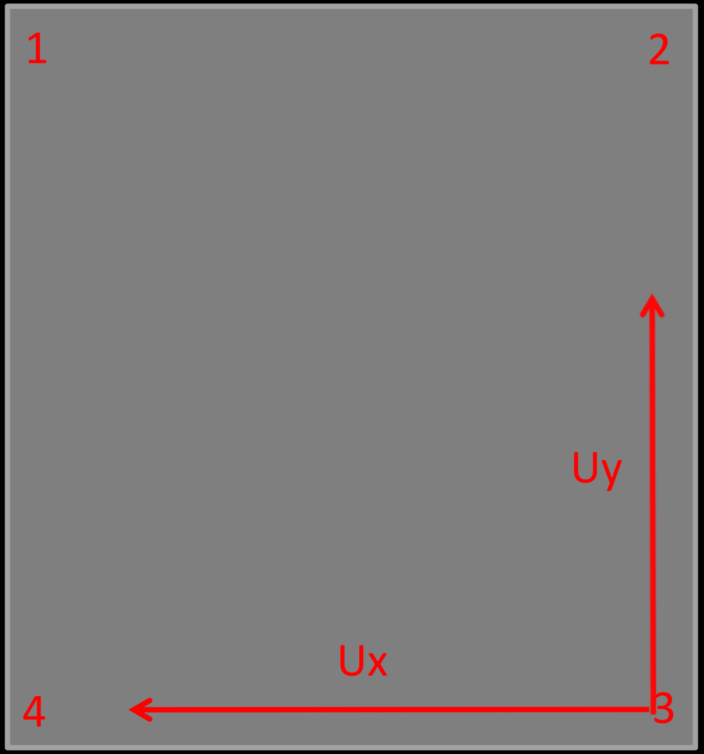


C3d File format specifies that **R** and O are supplied indirectly via the location of the four corners

$$\bar{U}_x = \frac{\bar{V}_x}{|V_x|}$$

$$\bar{U}_y = \frac{\bar{V}_y}{|V_y|}$$

$$\bar{U}_z = \bar{U}_x \times \bar{U}_y$$



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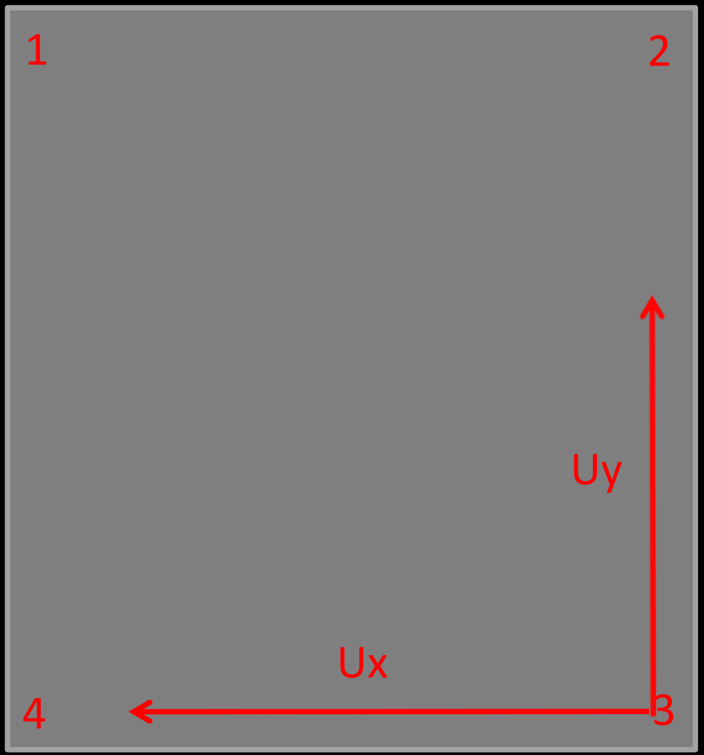
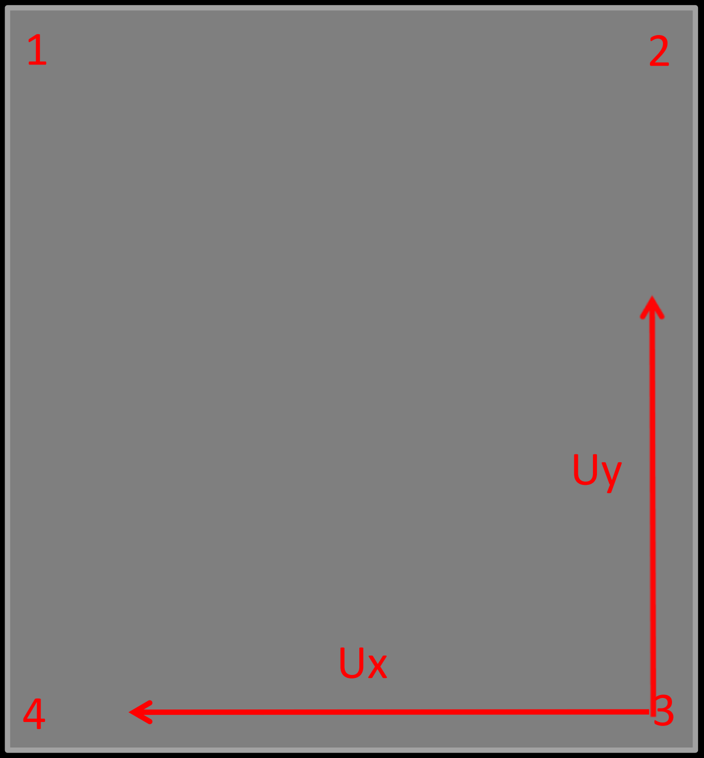


C3d File format specifies that **R** and O are supplied indirectly via the location of the four corners

$$R = \begin{vmatrix} Ux_x & Ux_Y & Ux_z \\ Uy_x & Uy_Y & Uy_z \\ Uz_x & Uz_Y & Uz_z \end{vmatrix}$$



Types of Force Platforms

Strain Gage:
- AMTI
- Bertec
- Kyowa-Dengyo

Piezo-Electric
- Kistler

Strain Gage Force Platforms

Strain Gage:

- Strain gauges are sensing devices that change resistance at their output terminals when stretched or compressed.

- length of conductor arranged in a zigzag pattern on a membrane

- Strain gauges are mounted in the same direction as the strain and often in fours to form a full 'Wheatstone Bridge’

Strain Gage Force Platforms

Wheatstone Bridge

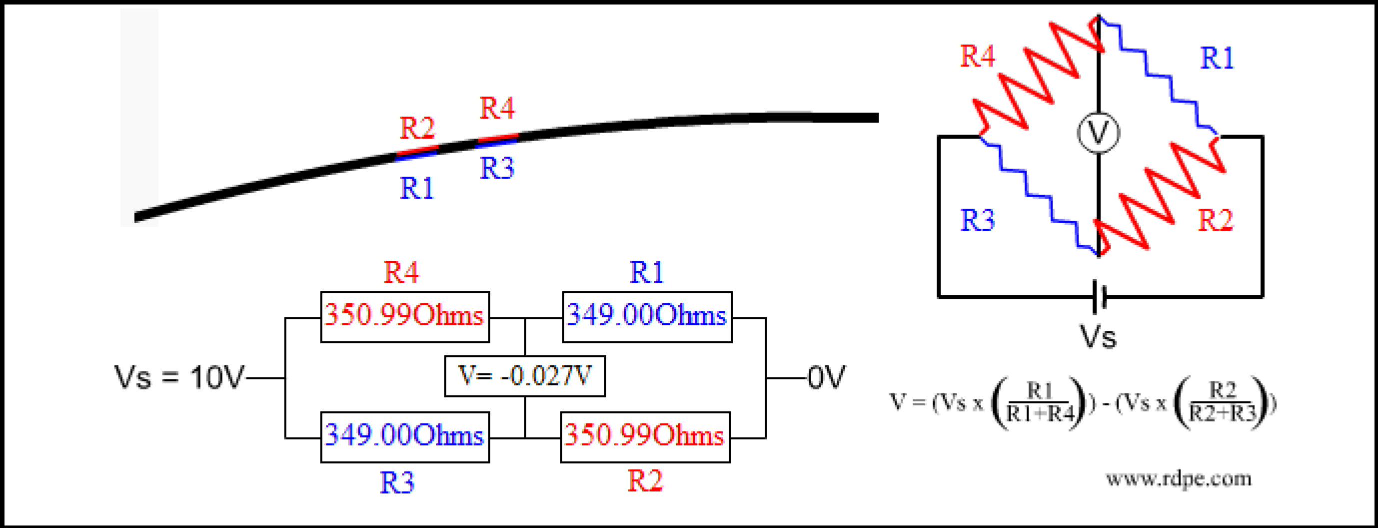


Image from RDP Electronics, Inc

Strain Gage Force Platforms

Strain Gage:

- when all four resistors are equal the bridge is balanced (Vout = 0 volts)

- Bridge Output Voltage:
  Voutput = Vexcitation * change in resistance

- Accurate measurements depend on a stable and low noise excitation source voltage

Strain Gage

Advantages:

- Good Accurarcy
- More available in custom sizes
- Cost

Disadvantages:

- Lower Frequency Response
- More limited force Range

For this HESC 689 we will concentrate on Strain Gage Force Platforms

AMTI

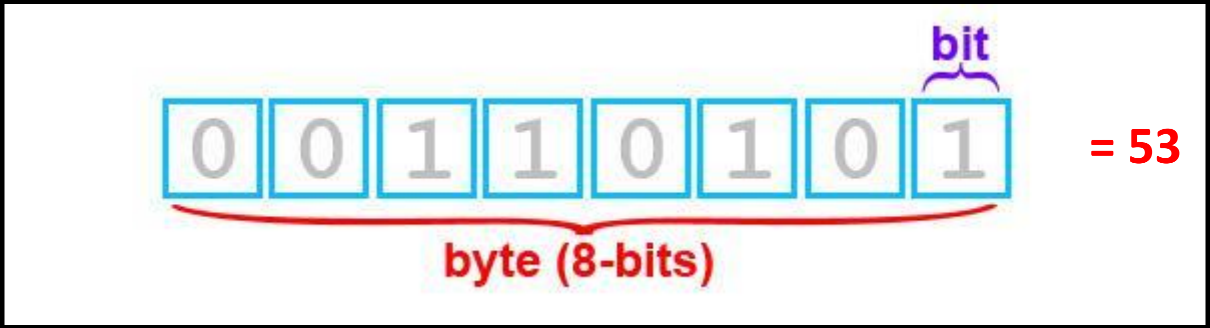
Bertec

Going from the force plate signals to obtain the Forces, Center of Pressure and Free Moment generated on the force platform

Background

Data from the force plate (plate force and plate moments) are converted from Analog to Digital using an A to D board

Analog to Digital Boards: Bits and Bytes



12 bit A to D board (2^{12}):

- Supplies values between 0 and 4095 (unsigned)

or

- Supplies values between -2047 and 2047 (signed)

Going from the force plate signals to obtain the Forces, Center of Pressure and Free Moment generated on the force platform

Step 1) Start by calculating the amplified output voltage for each channel

$$Va_i = (Digital_i - Digital\_Offset) * VoltageRange/(AtoDBoardRange)$$

Where:

Va_i is the amplified output voltage

$Digital_i$ is the value of the signal coming from the A to D board

Digital_Offset is the digital value corresponding to zero volts

Voltage Range is the Range of the AtoD board in Volts

For example for a 12 bit A to D (and offset of 0) board the equation becomes (Digital goes from -2047 to 2047):

$$Va_i = (Digital_i - 0) * VoltageRange/(4094)$$

If the voltage range was 10 Volts and the Signal was at the Max of the A to D board:

$$Va_i = \frac{(2047) * 10}{4094} = 5\,Volts$$

Sometimes you will see a 12 bit A to D where the digital output goes between Digital goes from 0 to 4095. In this case the offset Is usual set to 2048 and

$$Va_i = (Digital_i - 2048) * VoltageRange/(4095)$$

Step 2) However since each plate is calibrated using the output Voltage (V_i) not the amplified output voltage (Va_i) we need to Establish the relationship between the two.

Now each amplifier will have an excitation voltage ($V_{excitation}$) and each channel will have its own Gain (G_i) thus the force plate output voltage (V_i) can be related to the amplified output voltage (Va_i) by:

$$Va_i = V_{excitation} * G_i * V_i * 10^{-6}$$

where 10^{-6} is needed because the output of the force platform is in Newtons/microvolts (forces) or N*m/microvolts (moments)

From the last slide:

$$Va_i = V_{excitation} * G_i * V_i * 10^{-6}$$

Thus

$$V_i = \frac{Va_i}{V_{excitation} * G_i * 10^{-6}}$$

Substitute a new variable:

$$GF_i = \frac{1}{V_{excitation} * G_i * 10^{-6}}$$

Therefore the relationship between the output voltage and the amplified output voltage is:

$$V_i = GF_i \, Va_i$$

where:

$$GF_i = \frac{1}{V_{excitation} * G_i * 10^{-6}}$$

Example:

$$V_{excitation} = 10$$

$$G_i = 4000$$

Then:

$$GF_i = \frac{1}{V_{excitation} * G_i * 10^{-6}}$$

$$GF_i = \frac{1}{10 * 4000 * 10^{-6}} = 25$$

Note: Nexus 1.4.1 refers to the as the “Correction factor” and in Eva/RT or Cortex this value appears in the first line for each force plate in forepla.cal files

AMTI Strain Gage Amplifier

Provides:

- Strain Gage amplification
- Bridge Excitation
- Filters

Step 3) Compute the Force (Fx, Fy, Fz) and Moments (Mx, My, Mz) Coming from the plate

Each Strain Gage Manufacturer supplies a 6x6 Calibration matrix to compute the force on the plate so that:

$$F = CV_i$$

Where:

$$\begin{vmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{vmatrix} \begin{vmatrix} V_{i1} \\ V_{i2} \\ V_{i3} \\ V_{i4} \\ V_{i5} \\ V_{i6} \end{vmatrix}$$

However we use our A to D board to measure the amplified output voltage via*

$$Va_i = (Digital_i - 0) * VoltageRange/(4094)$$

And:

$$V_i = GF_i \, Va_i$$

Thus we convert the previous matrix multiplication:

$$\begin{vmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{vmatrix} \begin{vmatrix} GF_1 V_{a1} \\ GF_2 V_{a2} \\ GF_3 V_{a3} \\ GF_4 V_{a4} \\ GF_5 V_{a5} \\ GF_6 V_{a6} \end{vmatrix}$$

* Equation is for a 12 bit A to D board

Most off-diagonal terms are considered small and thus our Expression can be simplified

$$\begin{vmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{vmatrix} = \begin{vmatrix} C_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & C_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{vmatrix} \begin{vmatrix} GF_1 V_{a1} \\ GF_2 V_{a2} \\ GF_3 V_{a3} \\ GF_4 V_{a4} \\ GF_5 V_{a5} \\ GF_6 V_{a6} \end{vmatrix}$$

Via Matrix Multiplication (force, moment on the plate):

$$\begin{vmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{vmatrix} = \begin{vmatrix} C_{11} GF_1 V_{a1} \\ C_{22} GF_2 V_{a2} \\ C_{33} GF_3 V_{a3} \\ C_{44} GF_4 V_{a4} \\ C_{55} GF_5 V_{a5} \\ C_{66} GF_6 V_{a6} \end{vmatrix}$$

If we use $C_{i1} GF_i$ as our gain then the Calibration Matrix is not needed. (This is c3d Type 2 platform; Type 4 used the full Calibration Matrix)

Example (c3d Type 2)

Let’s assume we have a signal of 1 Digital Unit (Offset=0)

$$Va_i = \frac{(1-0)*10}{4094} = 0.002442\,Volts$$

And (note: Amplifier Gain was set to 2000):

$$GF_i = \frac{1}{10*2000*10^{-6}} = 50$$

Then:

$$V_i = GF_i\,Va_i$$

$$V_i = 50 \cdot 0.002442 = 0.1221$$

Example (c3d Type 2)

Thus a signal of 1 Digital Unit

$$V_i = 50 \cdot\ 0.002442 = 0.1221$$

This could be treated as scale factor such that

$$\begin{vmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{vmatrix} \begin{vmatrix} 0.1221 * D_1 \\ 0.1221 * D_2 \\ 0.1221 * D_3 \\ 0.1221 * D_4 \\ 0.1221 * D_5 \\ 0.1221 * D_6 \end{vmatrix}$$

Note: this is the force exerted on the plate not the Reaction Force

Example (c3d Type 2)

To get the "Reaction" Force and Moments

$$\begin{vmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{vmatrix} \begin{vmatrix} -0.1221 * D_1 \\ -0.1221 * D_2 \\ -0.1221 * D_3 \\ -0.1221 * D_4 \\ -0.1221 * D_5 \\ -0.1221 * D_6 \end{vmatrix}$$

Let's look at: FP4 Caltester Trial1.c3d

Force Platform Baseline (c3d parameter: FP_ZERO)

The FORCE_PLATFORM:ZERO parameter is an array that contains two non-zero signed integer values. These specify the range of 3D frames that may be used to provide a baseline for the force platform measurements

This allows any application that reads the force plate data to read in the analog data for the given frames, find the mean for each channel (V_{a_mean}), and subtract it from the analog data for the corresponding channel

Source: c3d file documentation maintained at www.c3d.org

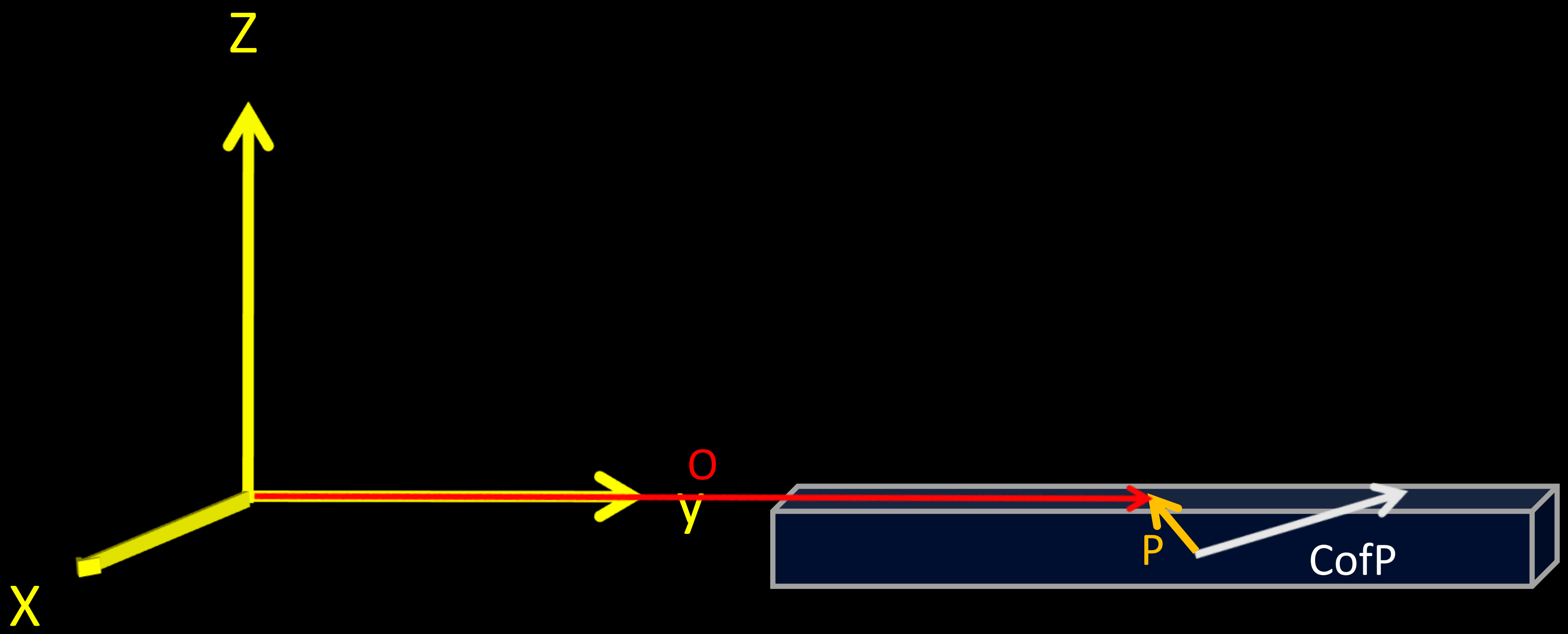
Force Platform Baseline (c3d parameter: FP_ZERO)

$$\begin{vmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{vmatrix} = \begin{vmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{vmatrix} \begin{vmatrix} GF_1 V_{a1} - GF_1 V_{a\_mean\ 1} \\ GF_2 V_{a2} - GF_2 V_{a\_mean\ 2} \\ GF_3 V_{a3} - GF_3 V_{a\_mean\ 3} \\ GF_4 V_{a4} - GF_4 V_{a\_mean\ 4} \\ GF_5 V_{a5} - GF_5 V_{a\_mean\ 5} \\ GF_6 V_{a6} - GF_6 V_{a\_mean\ 6} \end{vmatrix}$$

Computing force, center of pressure and free moment for a strain gage force platform (c3d file: type 2 or 4)

Location of the Plate Origin

Location of the origin of the force platform is dependent on the location of the force sensors but in general it will be located below the surface of the plate.



Outputs:

- Ground Reaction Force (F_x, F_y, F_z)*
- Plate Moments (M_x, M_y, M_z)

Calculated Values:

- Ground Reaction Force (F_x, F_y, F_z)*
- Center of Pressure ($CofP_x$, $CofP_y$, $CofP_z$)
- Free Moment (τ_x, τ_y, τ_z)

* Note the ground reaction force comes directly from the force platforms

Center of Pressure: Type 2

In general:

$$\bar{\tau} = \bar{R} \; x \; \bar{F}$$

For force platform:

$$\bar{M} = \overline{CofP} \; x \; \bar{F}$$

Center of Pressure: Type 2

For force platform:

$$\bar{M} = \overline{CofP}\ x\ \bar{F}$$

Via Cross product:

$$M_x = CofP_y \cdot F_z - CofP_z \cdot F_y$$

$$M_y = CofP_z \cdot F_x - CofP_x \cdot F_z$$

Now from the force platform specs:

$$fp\_origin_z = CofP_z$$

Center of Pressure: Type 2

Thus:

$$M_x = CofP_y \cdot F_z - fp\_origin_z \cdot F_y$$

$$M_y = fp\_origin_z \cdot F_x - CofP_x \cdot F_z$$

Via Algebra (CofP local):

$$CofP_x = \frac{fp\_origin_z \cdot F_x - M_y}{F_z}$$

$$CofP_y = \frac{fp\_origin_z \cdot F_y + M_x}{F_z}$$

$$CofP_z = fp\_origin_z$$

Free Moment:

$$M_z = \left(CofP_x \cdot F_y - CofP_y \cdot F_x\right) + FreeMoment$$

$$FreeMoment = M_z - (CofP_x \cdot F_y - CofP_y \cdot F_x)$$

Example:

Given:

$$fp\_origin = -0.004\hat{\imath} + 0.002\hat{\jmath} - 0.021\hat{k}$$

$$F_{local} = 25.3\hat{\imath} + 67.7\hat{\jmath} - 493.4\hat{k}$$

$$M_{local} = 16.0\hat{\imath} + -29.5\hat{\jmath} - 5.2\hat{k}$$

Find:

$$CofP_x = \frac{fp\_origin_z \cdot F_x - M_y}{F_z}$$

$$CofP_y = \frac{fp\_origin_z \cdot F_y + M_x}{F_z}$$

Example:

Answer CofP (local):

$$CofP_x = \frac{-0.021 \cdot 25.3 + 29.5}{-493.4} = -0.059$$

$$CofP_y = \frac{-0.021 \cdot 67.7 + 16.0}{-493.4} = -0.0294$$

$$CofP_z = -0.021$$

Example:

Free Moment (local):

$$FreeMoment = M_z - (CofP_x \cdot F_y - \ CofP_y \cdot F_x)$$

$$FreeMoment = -5.2 - (-0.059 \cdot 67.7 + 0.0294 \cdot 25.3) = -1.95$$

Putting it all together

PuttingItAllTogether.docx